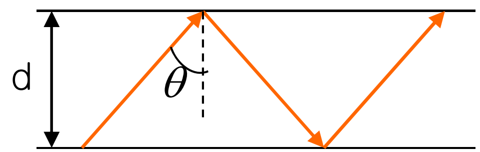
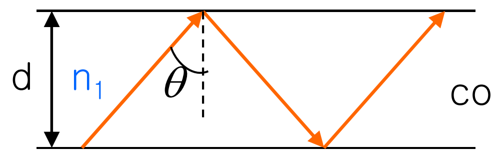
# Lect. 12: Dielectric Waveguide (1)

Guidance condition in a waveguide

Metallic waveguide




Dielectric waveguide (TIR)




$$e^{-jk_y d}(-1)e^{-jk_y d}(-1) = e^{-j2k_y d} = 1$$

$$\therefore 2k_y d = 2m\pi \text{ and } k_y = \frac{m\pi}{d}$$

$$\beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$$

$$e^{-jk_y d}\ r_{\perp,//}\ e^{-jk_y d}\ r_{\perp,//} = 1$$

$$\text{Since } r_{\perp,//} = e^{j\phi_{\perp,//}},\ e^{-j2k_y d} e^{j2\phi_{\perp,//}} = 1$$

$$\therefore 2k_y d - 2\phi_{\perp,//} = 2m\pi \text{ or } k_y d - \phi_{\perp,//} = m\pi$$

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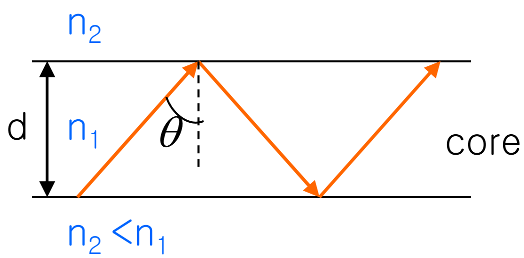


$$k_y d - \phi_{\perp,//} = m\pi$$

Remember

$$\phi_{\perp} : \ \tan\left(\frac{\phi_{\perp}}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i} \quad \text{(TE)}$$

$$\phi_{//} : \ \tan\left(\frac{\phi_{\parallel} + \pi}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i} \quad \text{(TM)}$$

Numerically solve for $k_y$ (TE, TM) , and then $\beta$

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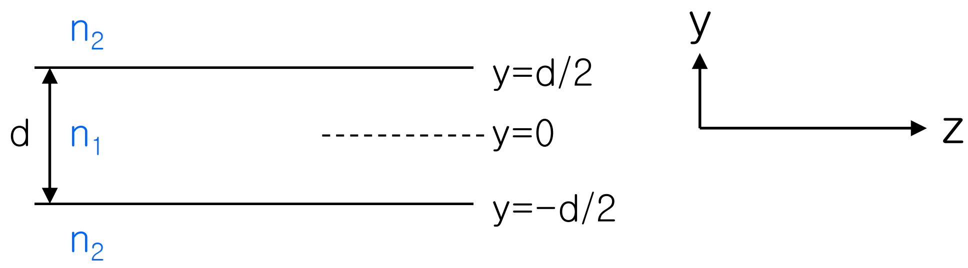


Full analysis starting from wave equations.

$$\nabla^2 \overline{E}(y,z,t) = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}(y,z,t)$$

Assuming $\overline{E}(y,z,t) = \overline{E}(y,z) \cdot e^{j\omega t}$,

$$\nabla^2 \overline{E} + k^2(y)\overline{E} = 0, \text{ where } k^2(y) = \mu\varepsilon(y)\omega^2$$

$$k(y) = n_2 k_0 \text{ for } |\mathrm{y}| > \frac{d}{2}; \text{ cladding}$$

$$k(y) = n_1 k_0 \text{ for } |\mathrm{y}| < \frac{d}{2}; \text{ core}$$

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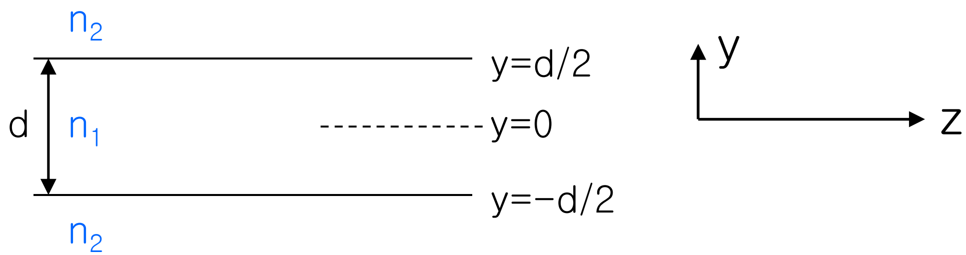


Consider TE Solution.

$$\bar{E}(y,z,t) = \bar{x}\, E(y)\, e^{-j\beta z}$$

Then, $$\frac{d^2E(y)}{dy^2} + (k^2(y) - \beta^2)E(y) = 0$$

=> Eigen value equation. Solve for $\beta$ and $E(y)$

$k^2(y) - \beta^2 > 0$ in core => $E(y) \sim \sin(k_y y)$ or $\cos(k_y y)$ with $k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$

$k^2(y) - \beta^2 < 0$ in cladding => $E(y) \sim \exp(\alpha y)$ or $\exp(-\alpha y)$ with $\alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$

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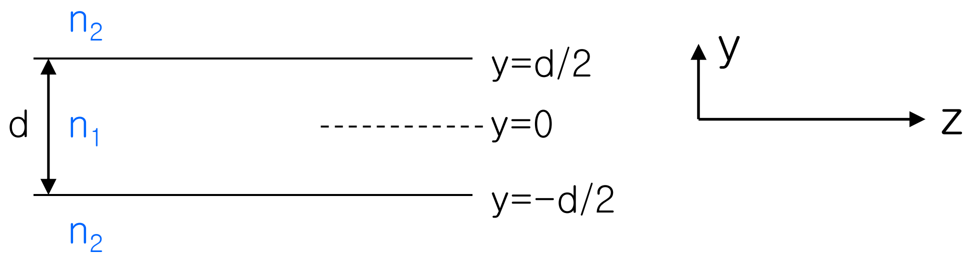


Solutions

$$y > \frac{d}{2} : E(y) = A\exp(\alpha y) + B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = C\sin(k_y y) + D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E\exp(\alpha y) + F\exp(-\alpha y)$$

Here, A=0 and F=0.

For easy analysis, divide the solutions into even and odd solutions

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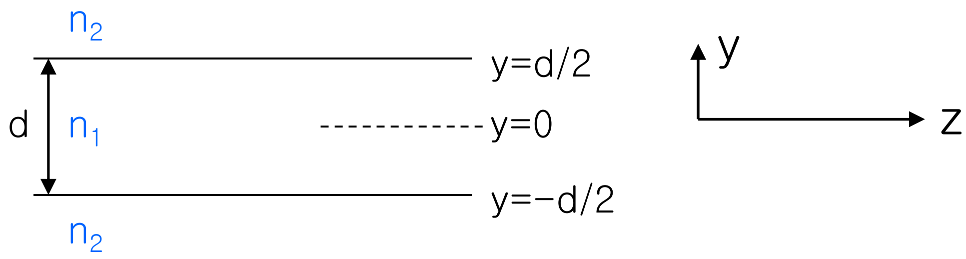


Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$(E = B)$

Apply boundary conditions:

$E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm\frac{d}{2}$

$$B\exp(-\alpha\frac{d}{2}) = D\cos(k_y\frac{d}{2}) \quad \text{------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = -k_y D\sin(k_y\frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = k_y\tan(k_y\frac{d}{2})$$

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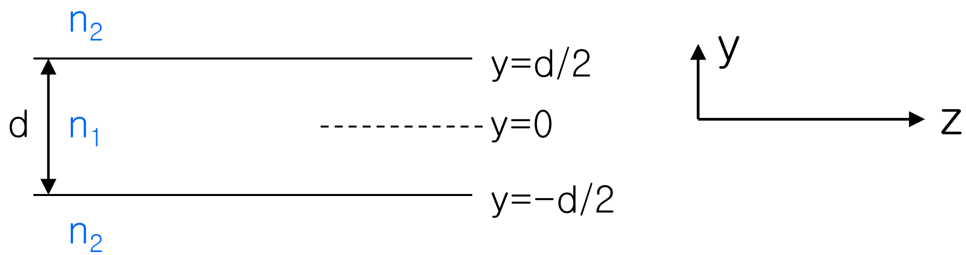



Odd Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$$(E = -B)$$

Apply boundary conditions.

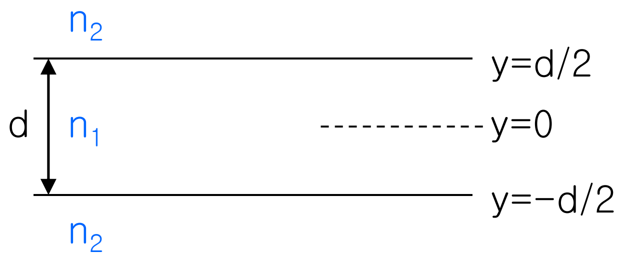
$E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm\frac{d}{2}$

$$B\exp(-\alpha\frac{d}{2}) = D\sin(k_y\frac{d}{2}) \quad \text{-------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = k_y D\cos(k_y\frac{d}{2}) \quad \text{------ (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = -k_y\cot(k_y\frac{d}{2}) = k_y\tan(k_y\frac{d}{2} - \frac{\pi}{2})$$

# Lect. 12: Dielectric Waveguide (1)





Even: $\alpha = k_y \tan(k_y \frac{d}{2})$

What do these mean?

Odd: $\alpha = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$

Determine $k_y$ and $\alpha$ that satisfy above conditions.

For graphical analysis, do following normalization.

Let $X = k_y \frac{d}{2}, \; Y = \alpha \frac{d}{2}$

Then, $Y = X \tan X$ for even

$Y = X \tan(X - \frac{\pi}{2})$ for odd

Plot these on X-Y plane.

$$\text{But } X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 [(n_1 k_0)^2 - \beta^2 + \beta^2 - (n_2 k_0)^2]$$

$$= \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2) = r^2$$

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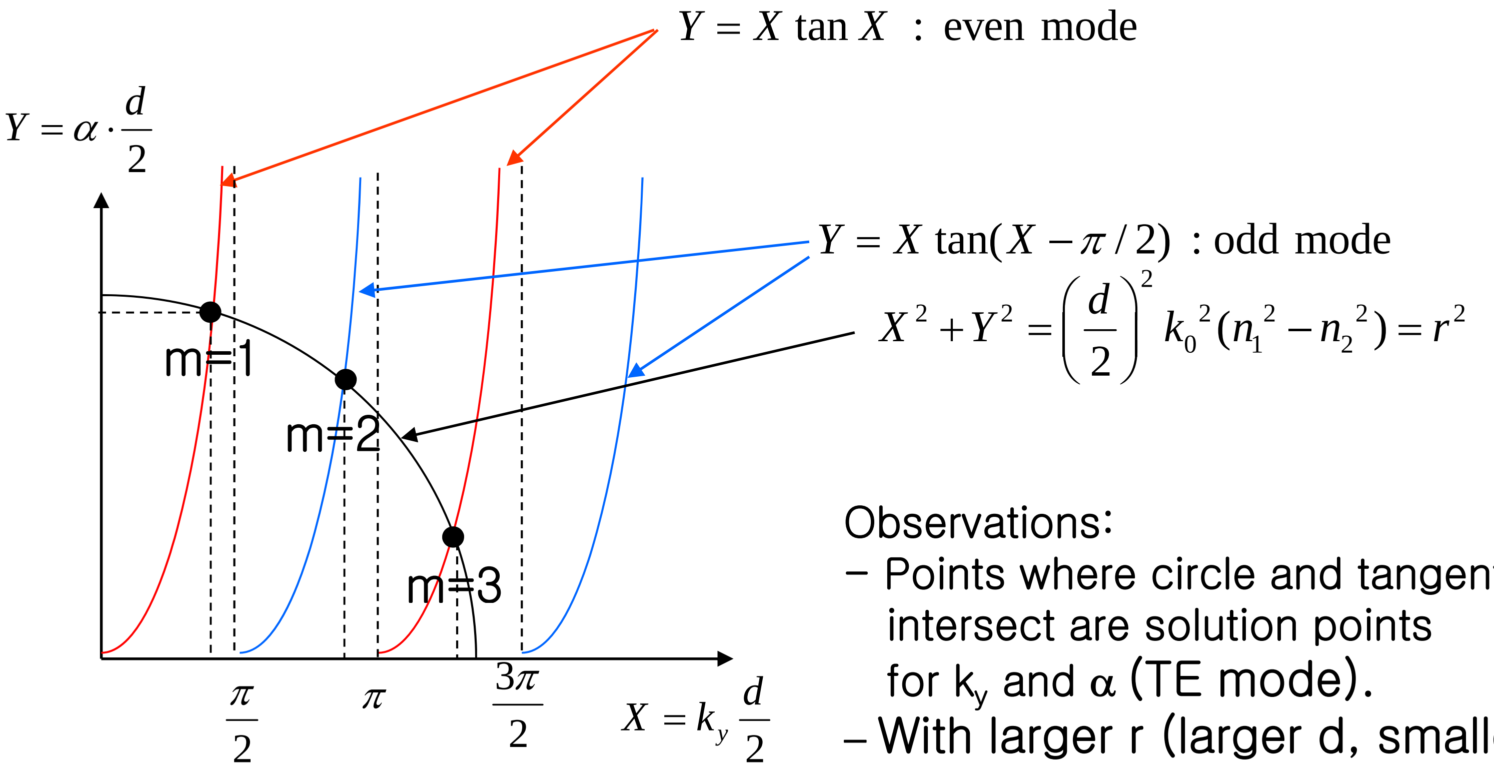


Observations:

- Points where circle and tangent curves intersect are solution points for $k_y$ and $\alpha$ (TE mode).
- With larger r (larger d, smaller $\lambda$, larger $n_1^2 - n_2^2$), more modes exist.
- There is at least one even mode.
- Even, odd, even, odd ...

# Lect. 12: Dielectric Waveguide (1)

Homework (1999 광전자 Test 1-2) Due 10/18

A symmetric three-layer waveguide is shown below. Consider only TE mode for this problem.

(a) Determine how many modes the waveguide can support for λ=1.0μm.

(b) Sketch the electric field intensity in the waveguide for each mode.

(c) When λ is increased, the number of modes the waveguide can support may change. What is the largest wavelength for which the mode number remains the same as what was obtained in (a)?

(d) Now the half of the waveguide is replaced with perfectly-conducting metal as shown below. Which modes among those determined in (a) can survive?